**Absolute and Conditional Convergence**

**AP\* Learning Objective:** Determine whether a series converges or diverges. (LO 4.1A)

**Overview:** The majority of tests you learn are tests for convergence of series of all positive terms. When varying signs appear in a series, a new condition must be considered. Fortunately, there exists a theorem which says if a series has absolute convergence, changing the sign of any number of its terms will not cause the altered series to diverge. However, if a series of all positive terms diverges, the alternating form of that series may converge. If this happens, the alternating series is said to converge conditionally.

**Content and Practice:**

 Recall, the Alternating Series Test requires three conditions to be met.

An alternating series,  will converge if:

1. each  is positive;

2.  for all , for some integer *N*;

3. .

Conditional convergence of an alternating series can be examined either by finding that the absolute form of the series diverges, and then applying the alternating series test, or by reversing that order.

This review will use the absolute test first as it is the natural order when testing endpoints of an interval of convergence of a power series. The ratio test is applied to find the open interval of absolute convergence, then the endpoints are testsed.

1. Determine if  converges absolutely, converges conditionally or diverges.

2. Determine if  converges absolutely, converges conditionally or diverges

If a power series has a finite interval of absolute convergence, each endpoint must be tested for absolute convergence, conditional convergence or divergence.

3. The interval of absolute convergence for  is . Determine the convergence or divergence of each endpoint.

**Additional Practice:**

 1. Determine if  converges absolutely, converges conditionally or diverges.

 2. Which of the following series converge conditionally, but not absolutely?

 I.  II.  III. 

1. II only (B) III only (C) I & II only (D) II & III only

 3. Find the interval of convergence of .

 (A)  (B)  (C)  (D) 

**Solutions:**

***Content & Practice*:**

1. The given series converges conditionally.

 Diverges by the p-series test.

 Alternating series test:  for 

  for 

  Converges as an alternating series.

2. The absolute series diverges by the  term test for divergence. Since  the series cannot converge as an alternating series.

 by L’Hospital’s rule.

3. If , . The series diverges. (Harmonic series)

If , . The series converges. (Alternating Harmonic series) The series is conditionally convergent at .

***Additional Practice*:**

1.  converges conditionally.

  diverges by the limit comparison test with .

 

 Alternating series test:  for 

  for 

 

2. (D) II & III only

I.  converges absolutely by p-series test.

 II.  is the alternating harmonic series which converges conditionally.

III.  converges conditionally since it diverges absolutely and converges as an alternating series.

  for , so  diverges by direct comparison with .

  for  meets all three conditions of the alternating series test.

3. (B) 



 At ,  Diverges (Harmonic series)

 At ,  Converges (Alternating harmonic series)