**Limit Comparison Test**

**AP\* Learning Objective:** Determine whether a series converges or diverges. (LO 4.1A)

**Overview:** There are times when some of the more efficient tests for convergence such as the p-series test, ratio test, direct comparison test or integral test cannot be applied. In those cases the limit comparison test may be tried. This test is usually used after considering some of the other tests simply because it is slightly more involved. The test has three possible outcomes and an appropriate comparison series must be chosen.

*Limit Comparison Test:* Suppose that  and  for all  (*N* a positive integer).

1. If ,  , then  and  both converge or both diverge.

2. If  and  converges, then  converges.

3. If  and  diverges, then  diverges.

It is most helpful to choose a series, , for which you can easily determine convergence or divergence.

**Content and Practice:**

When trying to determine if a series converges, quickly examine the simpler tests first. For example the  term test for divergence will quickly verify  diverges.

Now consider the series . The ratio test will produce a value of 1 which means the test is inconclusive. The integral of the expression is very difficult to antidifferentiate. The comparison test might work with a cleverly chosen function, but the end behavior model, which is often the easiest for comparison, has the inequality in the wrong direction. The series  converges but . So after a quick examination of other methods, one can try the limit comparison test using the end behavior model.



The limit of the ratio is positive and finite, so both series either converge or diverge. The series  converges by the p-series test, so  converges as well.

Being consistent in setting up the ratio for this test will make it easier to remember the three possible results of the limit of each ratio, and the conclusion to be drawn from that result. Put the series you are exploring in the numerator, and the series about which you have knowledge, in the denominator. Notice in the statement of the limit comparison test above, the last two parts of the test draw their conclusions based on what the  does.

1. Briefly explain why each test would be difficult to use to determine the behavior of the series .

a. Ratio test

b. Direct comparison test with 

c. Integral test

2. Use the limit comparison test on the series  to determine if it converges or diverges.

**Additional Practice:**

Which convergence test(s) would work on the following series?

(The behavior of each series is not given in the solution, just the test names.)

1. 

2. 

3. 

4. 

5. 

6. Use the limit comparison test to determine whether  converges or diverges.

7. Use the limit comparison test to determine whether  converges or diverges.

**Solutions:**

***Content & Practice*:**

1. a) The  for this series. The ratio test is inconclusive.

b) The terms of the given series are smaller than  for each *n*, but  diverges.

c) The  in the denominator makes antidifferentiation difficult, if not impossible.

2. For very large *n*, .



The resulting limit is positive and finite, and  diverges (harmonic series), therefore  also diverges by the limit comparison test.

***Additional Practice*:**

1. Ratio test (often good with factorials and exponentials)

2.  term test for divergence

3. Ratio test, limit comparison test with 

4. Ratio test, limit comparison test with 

5. Direct comparison test with  , Integral test, limit comparison test with 

6. Use the end behavior model  for .



The limit is finite and positive and  converges by the p-series test, so  also diverges by the limit comparison test.

7. Use .





The limit is positive and finite, and  diverges by the p-series test, so  also diverges by the limit comparison test.